Introduction to Stochastic Processes

Spatial Statistics

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Previous Concepts

Previous Concepts

Probability theory

Random variables

Stochastic Processes

Concepts of probability theory

Concepts

- **Stochastic phenomenon or experiment**: Process whose outcome is subject to uncertainty.
- **Sample space**: Set of all possible outcomes of an experiment.
- **Event**: Any collection of outcomes contained in the sample space.

Example

- **Experiment**: Three vehicles taking a particular freeway exit turns left (L) or right
  (R) at the end of the exit ramp.
- **Sample space**: $\Omega = \{LLL, LLR, LRL, LRR, RLL, RLR, RRL, RRR\}$
- **Event**: At most one vehicle turns left $\{LRR, RLR, RRL, RRR\}$ with probability 0.5.

Previous Concepts

Probability theory

Random variables

Stochastic Processes

Concepts of random variables

- **Random variable**: It is a function whose domain is the sample space and whose range is the set of real numbers.
- **Discrete random variable**: Possible values either constitute a finite set or else can be listed in an infinite sequence (e.g. number of family members).
- **Continuous random variable**: Possible values either constitute all numbers in a single interval on the number line or all in a disjoint union of such intervals. The probability of observing a specific quantity c is zero, $\Pr(X = c) = 0$ (e.g. luggage weight).

Discrete random variable example

- **Experiment**: Three vehicles taking a particular freeway exit turns left (L) or right
  (R) at the end of the exit ramp.
- **Sample space**: $\Omega = \{LLL, LLR, LRL, LRR, RLL, RLR, RRL, RRR\}$
- **Event**: At most one vehicle turns left $\{LRR, RLR, RRL, RRR\}$ with probability 0.5.
- **Random** variable: Number of vehicles that turn left.
  $X(LLL) = 3$, $X(LLR) = 2$, $X(LRL) = 2$, $\ldots$, $X(RRR) = 0$.

Continuous random variable example

- **Experiment**: Evaluate the luggage weight of the passengers in particular flight.
- **Sample space**: $\Omega = < 0, +\infty >$
- *Event*: Receive a luggage with weight between 15 kg and 20 kg with probability $\Pr(X \in < 15, 20 >)$.
- **Random variable**: Luggage weight.

Stochastic Processes

Previous Concepts

Stochastic Processes

- Definition
- Examples

Stochastic process

A **stochastic process** is a collection or family $\{X(t) : t \in T\}$ of random variables mapping the sample space Ω to a set S. Characterization of the process will depending on the choices for:

- Index set (T): discrete or continuous.
- State space (S): discrete or continuous.

Example: Discrete-time stochastic process with continuous state-space.

Some examples for T and S are:

- $T = 0, 1, 2$
- $T = [0, \infty)$
- $S = \mathcal{Z}$
- $S = \mathcal{R}$

Spatial and/or temporal stochastic process

With students.

Stochastic process

In general, $X(t)$ are not independent and we are interested in studying the properties of the process as a whole. For example, if $S \subseteq \mathcal{R}$ and $T = 1, 2, \ldots, n$, then the vector $vex = (X(1), X(2), \ldots, X(n))$ comes from a joint distribution $F(\boldsymbol{x}) = \mathcal{P}(X(1) \leq x_1, X(2) \leq x_2, \ldots, X(n) \leq x_n)$.

Previous Concepts

Stochastic Processes

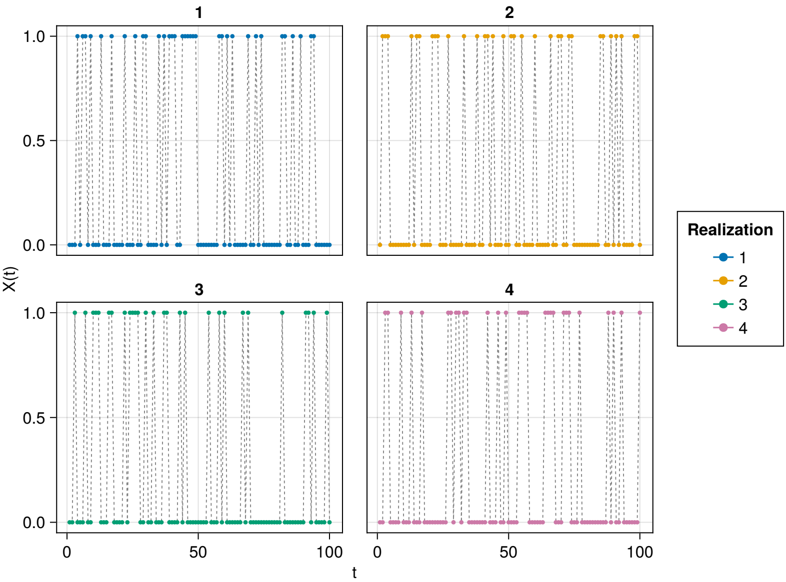
- Definition
- Examples

Bernoulli process

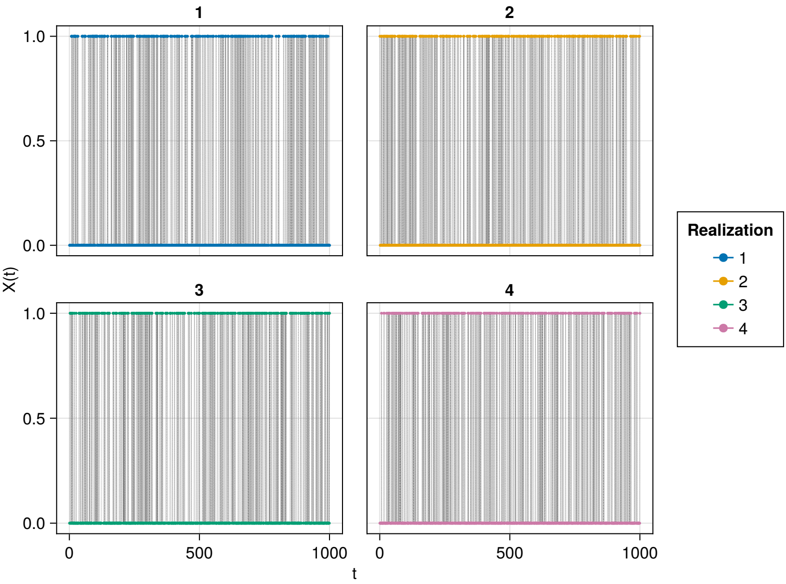
Let $\{X(t) : t \in \{1, 2, \ldots, T\}\}$ such as

$$X(t) \overset{iid}{\sim} Bernoulli(\pi).$$

Bernoulli process ($t = 10^2$)



Bernoulli process ($t = 10^3$)

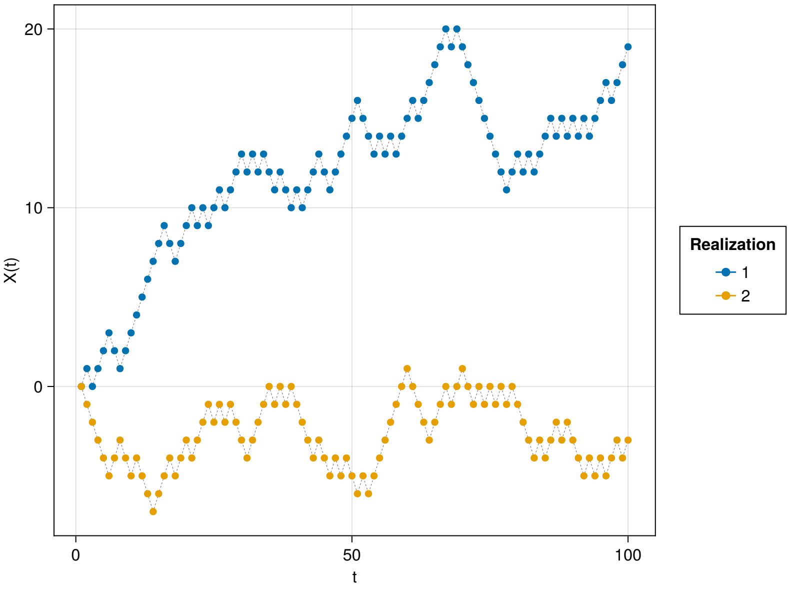


1D random walk

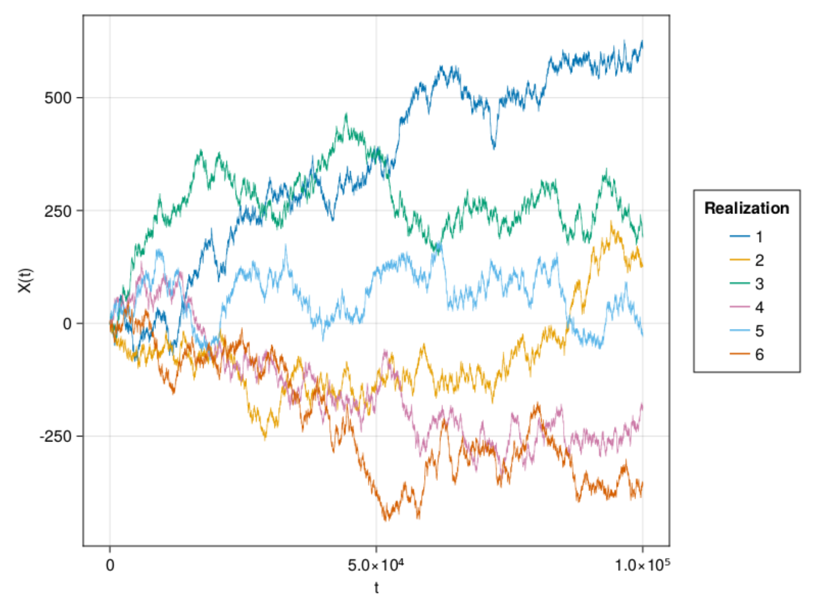
Let $\{X(t) : t \in \{1, 2, \ldots, T\}\}$ such as

$$
\begin{aligned}
X(1) &= 0, \\
X(t) &= X(t-1) + Z(t), \quad \text{for} \quad t = 2, \ldots, T. \\
Z(t) &= \begin{cases} 1, & \text{with probability } \pi \\ -1, & \text{with probability } 1 - \pi \end{cases}
\end{aligned}
$$

1D random walk ($T = 10^2$)



1D random walk ($T = 10^5$)

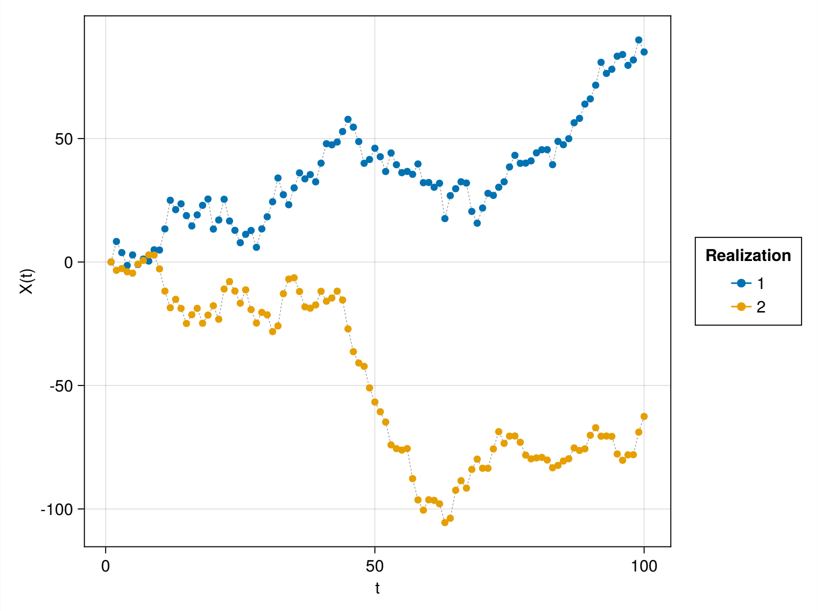


1D Gaussian random walk

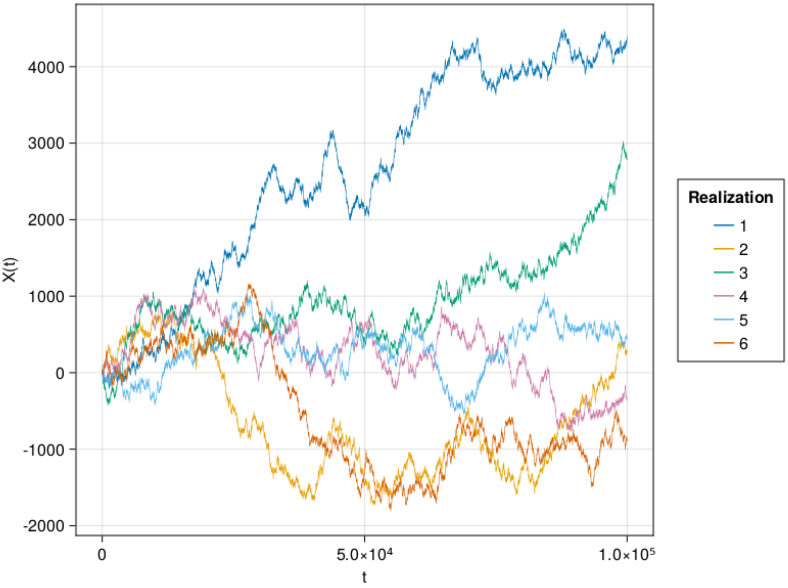
Let $\{X(t) : t \in \{1, 2, \ldots, T\}\}$ such as

$$\begin{aligned} X(1) &= 0, \\ X(t) &= X(t-1) + Z(t), \quad \text{for} \;\; t = 2, \ldots, T. \\ Z(t) &= Normal(\mu = 0, \sigma^2 = 1) \end{aligned}$$

1D Gaussian random walk ($T = 10^2$)



1D Gaussian random walk ($T = 10^5$)



2D Gaussian random walk

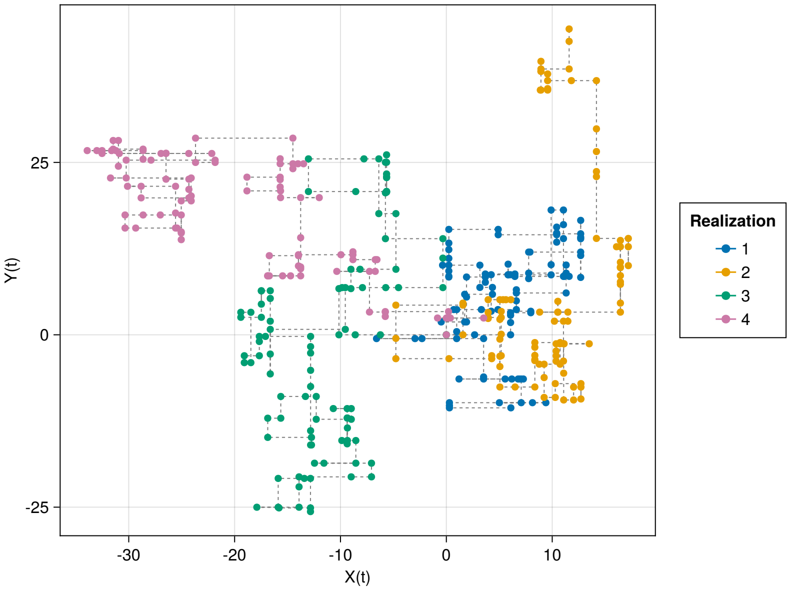
Let $\{\boldsymbol{X}(t) = (X_1(t), X_2(t)) : t \in \{1, 2, \ldots, T\}\}$ such as

$$\boldsymbol{X}(1) = (0, 0)$$

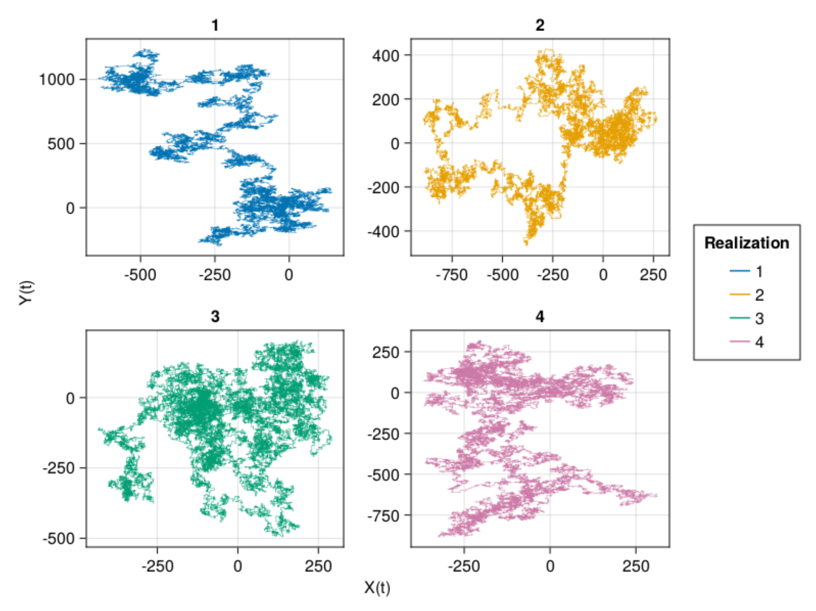
$$\boldsymbol{X}(t) = \begin{cases} (X_1(t-1), X_2(t-1) + Z(t)) & \text{with probability } 0.5 \\ (X_1(t-1) + Z(t), X_2(t-1)) & \text{with probability } 0.5 \end{cases}$$

$$Z(t) = Normal(\mu = 0, \sigma^2 = 9)$$

2D Gaussian random walk ($T = 10^2$)



2D Gaussian random walk ($T = 10^5$)

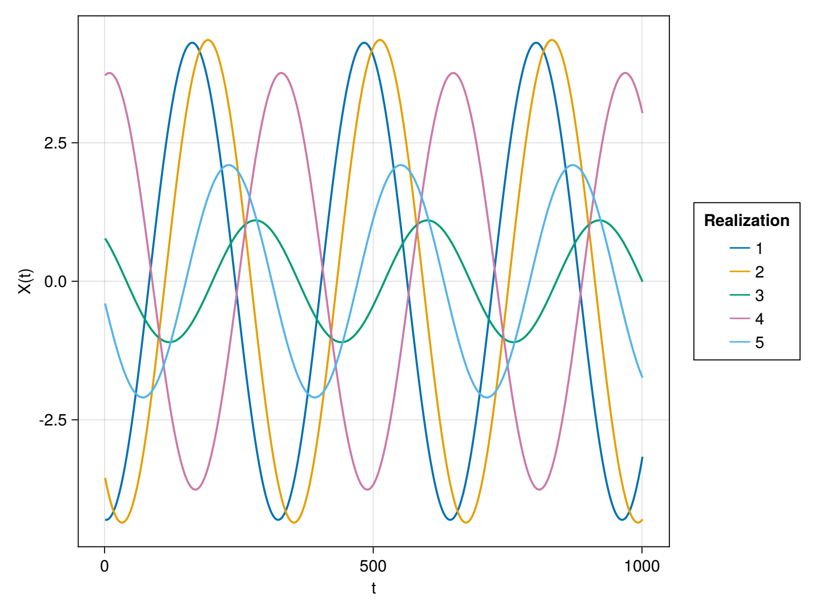


Sine and Cosine

Let $\{X(t) : t \in [0, T]\}$ such as

$$X(t) = Z_1 \times cos(t\lambda) + Z_2 \times sin(t\lambda),$$
$$Z_1 = Normal(\mu = 0, \sigma^2 = 9),$$
$$Z_2 = Normal(\mu = 0, \sigma^2 = 9).$$

Sin and Cos ($T = 10^2$)



Sin and Cos (T = 10^5)